VERY MAD FAMILIES.

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In this talk we will present a class of maximal almost disjoint (mad) families in the Baire space $\mathbb{N}^{\mathbb{N}}$.

DEFINITION. $f_1, f_2 \in \mathbb{N}^{\mathbb{N}}$ are *almost disjoint* iff

$$|f_1 \cap f_2| = |\{n \in \mathbb{N} \mid f_1(n) = f_2(n)\}| = \aleph_0.$$

A family $\mathcal{A} \subseteq \mathbb{N}^{\mathbb{N}}$ is *almost disjoint* iff every two functions in $\mathcal{A}$ are almost disjoint.

An $f \in \mathbb{N}^{\mathbb{N}}$ is *not finitely covered* by an $\mathcal{A} \subseteq \mathbb{N}^{\mathbb{N}}$ iff for every finite set of functions $g_1, \ldots, g_n \in A$ the set $f \setminus \bigcup_{1 \leq i \leq n} g_i$ is infinite. A family $F \subseteq \mathbb{N}^{\mathbb{N}}$ is *not finitely covered* by an $\mathcal{A} \subseteq \mathbb{N}^{\mathbb{N}}$ iff all $f \in F$ are not finitely covered by $\mathcal{A}$.

A family $\mathcal{A} \subseteq \mathbb{N}^{\mathbb{N}}$ is *strongly mad* iff for every countable $F \subseteq \mathbb{N}^{\mathbb{N}}$ that is not finitely covered by $\mathcal{A}$, there is a $g \in \mathcal{A}$ such that for all $f \in F$ the set $f \cap g$ is infinite.

A family $\mathcal{A} \subseteq \mathbb{N}^{\mathbb{N}}$ is *very mad* iff for every $F \subseteq \mathbb{N}^{\mathbb{N}}$ satisfying $|F| < |\mathcal{A}|$ and F is not finitely covered by $\mathcal{A}$, there is a $g \in \mathcal{A}$ such that for all $f \in F$ the set $f \cap g$ is infinite.

NOTE. The notion of strongly mad family was introduced by Juris Steprāns to study the question (asked by Yi Zhang) whether there are analytic mad families (the question is still open).

Obviously the notion of very mad family is a strengthening of the notion of strongly mad family. Also obviously, strongly mad families are mad.

We will present several basic results about the cardinality of very mad families, in particular we will outline the proof of the following theorem.

THEOREM. *If M is a model of* ZFC*, and in M we have a regular cardinal κ such that $\aleph_1 \leq \kappa \leq 2^{\aleph_0} = \lambda$, then there is a c.c.c. forcing $\mathbb{P}$ such that the model $M^{\mathbb{P}}$ of* ZFC *satisfies:*

- *There exists a very mad family $\mathcal{A}$ of cardinality κ and $\lambda = 2^{\aleph_0}$.*

We will also ask several open questions concerning these two types of families.

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