Separating Notions of Randomness

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Outline

- Definitions and Basics.
- Motivation.
- The Result and Idea.

Work joint with Steffen Lempp.

What is a random real?

- Null sets,
- Bit patterns,
- Betting against the real.

Martingale:

$$f : 2^{<\mathbb{N}} \to \mathbb{R}_0^+$$

such that

$$f(\sigma) = \frac{f(\sigma 1) + f(\sigma 0)}{2}$$

Success of martingale: unbounded capital growth (limsup).

(partial) Computable martingale:

$$f : 2^{<\mathbb{N}} \to \mathbb{Q}_0^+ \text{ (partial) computable.}$$

Effective martingale:

f such that there is a computable function $g : 2^{<\mathbb{N}} \times \mathbb{N} \to \mathbb{Q}_0^+$ increasing in the second coordinate with $f(\sigma) = \lim_{n\to\infty} g(\sigma, n)$.

A real $A \in 2^{\mathbb{N}}$ is X random: if no X martingale succeeds on A.

Computably random.

Partial computably random.

Martin-Löf random.

Theorem (Ambos-Spies)

$\mathrm{CR} \supsetneq \mathrm{PCR}$.

Theorem (combination Muchnik and Schnorr)

$\mathrm{PCR} \supsetneq \mathrm{ML}$.

All martingales so far monotonic.

selection rule: $s : (\mathbb{N} \times \{0,1\})^{<\mathbb{N}} \to \mathbb{N}$.

stake function: $q : (\mathbb{N} \times \{0,1\})^{<\mathbb{N}} \to [0,2]$.

Kolmogorov-Loveland random.

$$\mathrm{PCR} \supseteq \mathrm{KL} \supseteq \mathrm{ML}.$$

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$$\mathrm{KL} \stackrel{?}{=} \mathrm{ML}$$

Restrictions on s — determine s from some function h:

$$s(\tau) = h(|\tau|).$$

Theorem (Muchnik)

$\mathrm{PCR} \supsetneq$ *partial permutation randomness.*

Theorem (K. and Lempp)

Partial permutation randomness $\supsetneq \mathrm{ML}$*.*

Strategy:

"Find" real A and ML martingale M such that A is partial permutation random but M succeeds on A.

"Find" a single martingale that ensures the partial permutation randomness.

Beat it.

f and g martingales, $f + g$ and cf martingales.

For monotonic only!

Monotenize

$$d^{\text{expec}}_{\langle h,q \rangle}(\sigma) :=$$

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$$d^{\text{expec}}_{\langle h,q\rangle}(\sigma) := \sum_{\substack{p \in 2^{l_\sigma} \\ \sigma \prec p}} d^{p}_{\langle h,q\rangle}(n_\sigma) 2^{-(l_\sigma - |\sigma|)}$$

Partiality

$\langle (h_i, q_i) : i \in \mathbb{N} \rangle$ enumeration of all partial permutation martingales.

Sum of earlier permutation gales related to the new one.

New one diverges where sum is small, ... or not.

f total monotonic, (h_i, q_i) next partial permutation martingale to look at.

Make total by betting even.

sum different E_x with appropriate scaling factors.

$$1 = \sum_{n=1}^{\infty} \frac{1}{2^n}$$

Add a new E_x if some strategy succeeds; if the x become long enough the capital grows without bound.

Theorem (K. and Lempp)

Partial permutation randomness $\supsetneq \mathrm{ML}$.

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Center for Dynamics and Geometry Seminar

Generating Sets of Cofinitary Groups.

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Time:	03:35pm - 04:35pm